Comment on "Convective and Radiative Heat Transfer to an Ablating Body"

H. T. YANG*
University of Southern California, Los Angeles, Calif.

THE treatment of the momentum equation in Ref. 1 is identical to that employed in Ref. 2, where additional details are presented. In this regard, boundary conditions are applied at the vehicle wall and immediately behind the shock. The shock-boundary conditions used are the usual Rankine-Hugoniot relations expressed by Eqs. (23) of Ref. 2.

The validity of these boundary conditions appears to be questionable for the following reasons. Physically, the Rankine-Hugoniot relations are valid only if the flowfield downstream as well as upstream of the shock is in translational equilibrium. Since the flowfield behind the shock is assumed viscous in Refs. 1 and 2, it is out of translational equilibrium. The Rankine-Hugoniot relations are therefore not strictly applicable here. Mathematically, Bush,[3] as well as Cheng,[4] has already ruled out the "viscous layer" regime, in which the viscous shock-layer equations are solved subject to the Rankine-Hugoniot relations at the outer edge.

A clarification of this point seems to be desirable.

References

[1] Hoshizaki, H. and Lasher, L. E., "Convective and Radiative Heat Transfer to an Ablating Body," *AIAA Journal*, Vol. 6, No. 8, Aug. 1968, pp. 1441–1449.

[2] Hoshizaki, H. and Wilson, K. H., "Convective and Radiative Heat Transfer During Superorbital Entry," *AIAA Journal*, Vol. 5, No. 1, Jan. 1967, pp. 25–35.

[3] Bush, W. B., "On the Viscous Hypersonic Blunt Body Problem," *Journal of Fluid Mechanics*, Vol. 20, Pt. 3, 1964, pp. 353–367.

[4] Cheng, H. K., "Hypersonic Shock-Layer Theory of the Stagnation Region at Low Reynolds Number," *Proceedings of the 1961 Heat Transfer and Fluid Mechanics Institute*, Stanford University Press, 1961, pp. 161–175.

Received August 14, 1968.
* Associate Professor. Associate Fellow AIAA.

Reply by Authors to H. T. Yang

H. HOSHIZAKI* AND L. E. LASHER†
Lockheed Missiles & Space Company, Sunnyvale, Calif.

PROFESSOR Yang has brought up an interesting and valid point on the use of inviscid boundary conditions in a viscous shock layer. He points out that "the Rankine-Hugoniot relations are therefore not strictly applicable here." The authors are well aware of this point (see Ref. 1); their reasons for considering the shock layer to be completely viscous in the subject paper were quoted in a previous paper (Yang's Ref. 2):

> An important physical mechanism in the shock-layer flow is the coupling between the inviscid and viscous regions created by radiant energy transfer. This coupling is taken into account by considering the entire flow region between the shock and the body simultaneously. The advantage of this approach is that it eliminates the necessity of matching the frequency dependent radiation flux at the inviscid-viscous interface.

The shock layer is considered to be viscous to avoid the tedious matching of the radiative flux, not because of low Reynolds number effects. In fact the Reynolds numbers are of the order of 10^5 to 10^6, based on nose radius, freestream velocities, and flow properties behind the shock.

Although the derivation of the thin shock-layer equations is based on the assumption that the shock layer is completely viscous, these equations are also applicable when the outer portions of the shock layer are nearly inviscid. The numerical results presented in the subject paper would not be significantly affected by the use of the proper viscous boundary conditions.

References

[1] Hoshizaki, H., "Shock-Generated Vorticity Effects at Low Reynolds Numbers," LMSC-48381 (ASTIA 210401), Vol. 1, Jan. 1959, Lockheed Missiles & Space Div., Sunnyvale, Calif., pp. 9–43.

[2] Levinsky, E. S. and Yoshihara, H., "Rarefied Hypersonic Flow Over a Sphere," Paper 1967-61, ARS-AFOSR International Hypersonics Conference, MIT, Aug., 16–18, 1961.

Received October 28, 1968.
* Senior Staff Engineer. Member AIAA.
† Research Scientist.

Comment on "Radiation from Conical Surfaces with Nonuniform Radiosity"

D. K. EDWARDS*
TRW Systems Group, Redondo Beach, Calif.

BOBCO[1] presents two graphs giving the shape factor from a receiver to a conical frustrum, shown in Fig. 1 and in Figs. 2–9. He indicates that these graphs can be fit with an exponential and that they can then be differentiated and inte-

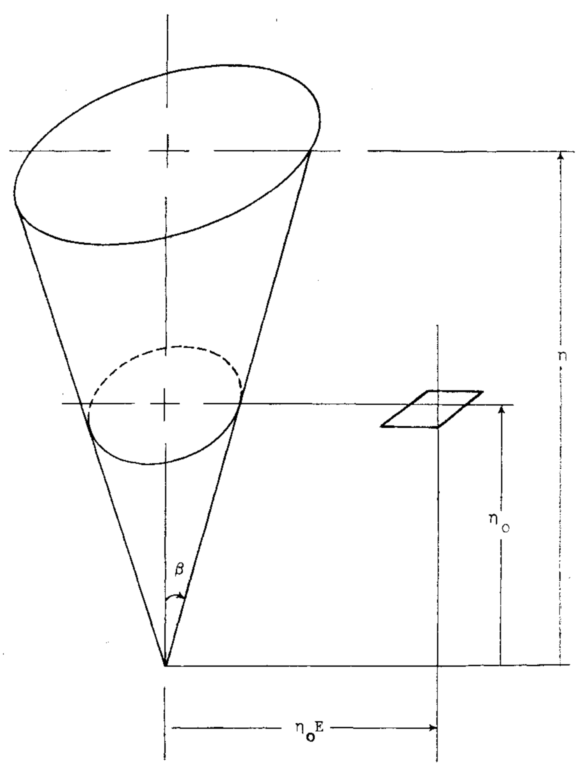

Fig. 1 Cone and receiver.

Received September 20, 1968.
* Consultant; also Professor of Engineering, University of California Los Angeles, Los Angeles, Calif. Member AIAA.

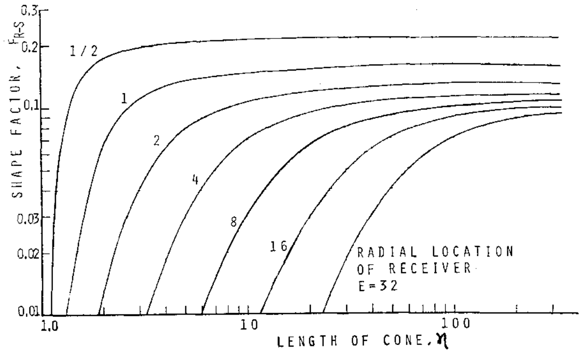

Fig. 2 Shape factor subtended by a 10° half-angle cone; $\omega = 0$, $\eta_0 = 1.0$, $\beta = 10°$.

grated with a nonuniform radiosity weighting factor to estimate radiant fluxes incident on surfaces facing aft near a rocket exhaust plume. Distributions of interest were indicated to be

$$J = J_0(\eta_0/\eta)^{\omega},\ \omega = 1,2,3 \qquad (1)$$

It has been discovered that the curves given by Bobco are erroneous for low values of E, that is, for receiver locations close to the cone. Of more interest, however, is the question of whether the curves, if corrected, would be of any practical importance, and, if so, under what conditions and for what values of ω. It is the purpose of this comment to present corrected and more extensive results and to argue that the Bobco concept of a cone with the nonuniform radiosity is indeed a useful concept, particularly for metallized solid propellent motors.

The first question to be answered is whether it is reasonable to approximate a rocket plume by an opaque conical surface. In order to do so the plume must be optically thick with a well-defined boundary. At high altitudes the gases in the underexpanded exhaust rapidly expand and cool to leave a core of metal oxide particles. Those particles appreciably less than 1 μ in size change trajectory somewhat with the gas and cool rather rapidly. However, for aluminized propellent engines with nozzle throat diameters on the order of 1 in., such as the Aerojet SVM-2 engine used on the TRW-INTELSAT III vehicle, this cloud of light particles is virtually nonabsorbing and thin optically. Those particles 1 μ and larger in size have their trajectories affected relatively little in the near flowfield by the expanding gas, and do form a fairly well-defined optically scattering-thick conical plume. Figure 7 of Morizumi and Carpenter[2] shows such a cone for the engine mentioned previously. The optical depth based on geometric cross section for this engine is approximately 3.68.[3] The optical-scattering depth is approximately the same, because in the large particle limit the scattering efficiency Q_s goes to 2.0, but the diffracted radiation should not be included,[4] so that the effective Q_s is 1.0. Since exp

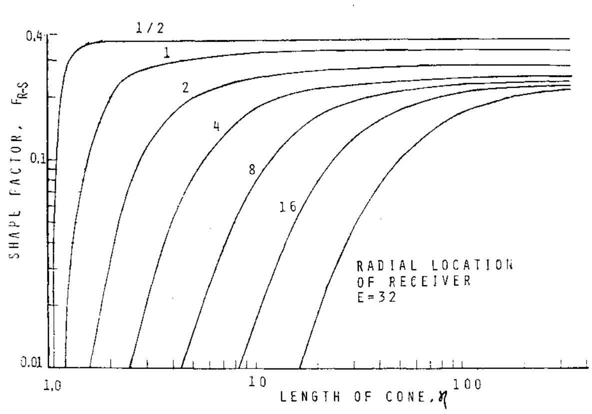

Fig. 3 Shape factor subtended by a 20° half-angle cone; $\omega = 0$, $\eta_0 = 1.0$, $\beta = 20°$.

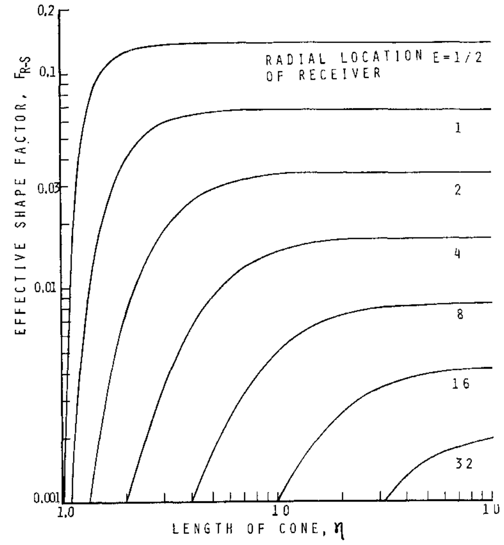

Fig. 4 Effective shape factor subtended by a 10° half-angle cone with nonuniform radiosity; $\omega = 1.0$, $\eta_0 = 1.0$, $\beta = 10°$.

(-3.68) is small, the plume can be considered thick. It would appear from Morizumi and Carpenter's figure that the nozzle cone angle $\beta = 19°$ defines an appropriate cone.

The second question to be answered is what value of ω might be appropriate. For an engine with 1 μ and larger alumina particles, an appreciable distance is required for the particles to solidify. Thus, the particle temperature remains relatively constant. If the conical cloud of particles were optically absorbing-thick, the value $\omega = 0$ would be appropriate. However, Morizumi and Carpenter are very likely in error when they propose a value of particle emissivity or absorption efficiency $Q_a = 0.25$ for Al_2O_3. It appears from the work of Plass[5] that the value must vary significantly with wavelength and must be appreciably lower than 0.25. Experiment indicates an average value of $Q_a = 0.02$ is about correct, although Plass' calculations indicate a still lower value. Under these conditions the plume is nearly absorbing-thin, so that a value of $\omega = 1$ is appropriate, since the plume density is decreasing with $1/\eta^2$ and the cone circumference is increasing like η.

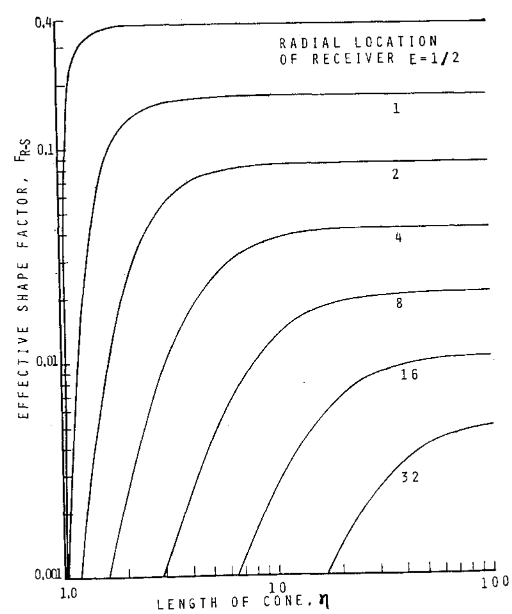

Fig. 5 Effective shape factor subtended by a 20° half-angle cone with nonuniform radiosity; $\omega = 1.0$, $\eta_0 = 1.0$, $\beta = 20°$.

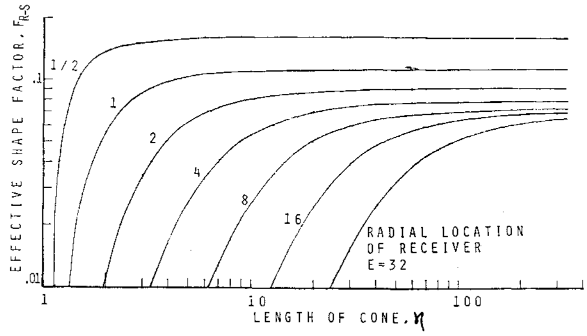

Fig. 6 Effective shape factor subtended by a 10° half-angle cone with nondiffuse emittance; ϵ/ϵ normal = $0.5 + 0.5 \cos\theta$, $\omega = 0$, $\eta_0 = 1.0$, $\beta = 10°$.

A third question to be answered is whether the plume radiosity should be regarded as diffuse. Bobco may be interpreted to imply that the plume is diffuse, for he makes no provision for a directionally varying emissivity in Ref. 1. The H function of Chandrasekhar[6] may be used to derive the directional emissivity (via the bidirectional reflectivity) of a scattering bed with single particle albedo $Q_s/(Q_a + Q_s)$. For low albedo the emissivity tends to be diffuse, whereas for high albedo it decreases significantly with angle from the normal. Therefore, it is not reasonable to assume the plume radiosity to be perfectly diffuse for an aluminized propellent.

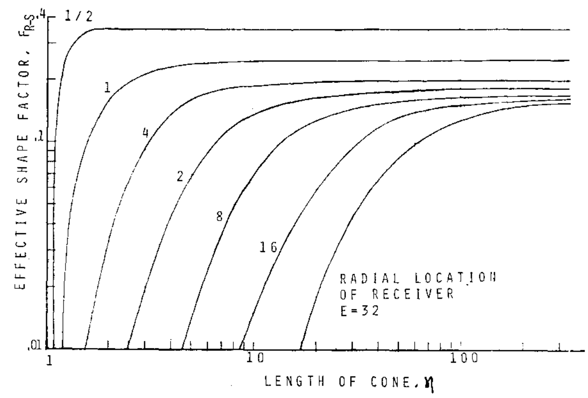

Fig. 7 Effective shape factor subtended by a 20° half-angle cone with nondiffuse emittance; ϵ/ϵ normal = $0.5 + 0.5 \cos\theta$, $\omega = 0$, $\eta_0 = 1.0$, $\beta = 20°$.

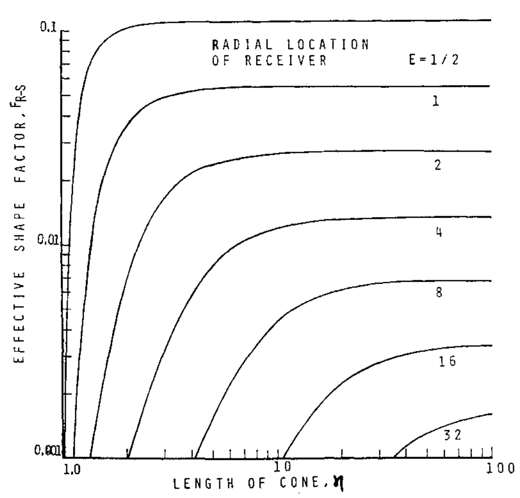

Fig. 8 Effective shape factor subtended by a 10° half-angle cone with nonuniform radiosity and nondiffuse emittance; ϵ/ϵ normal = $0.5 + 0.5 \cos\theta$, $\omega = 1.0$, $\eta_0 = 1.0$, $\beta = 10°$.

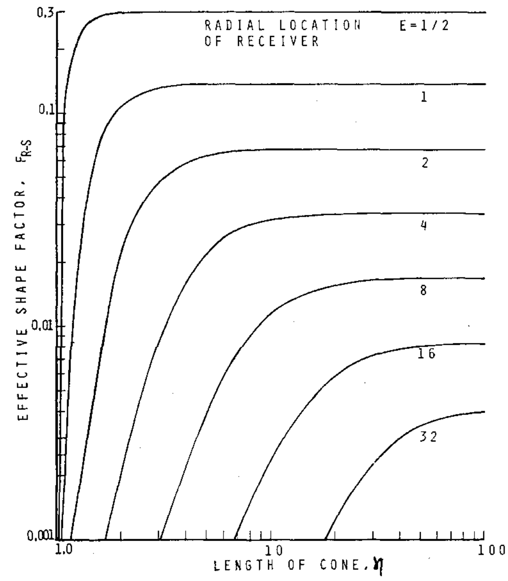

Fig. 9 Effective shape factor subtended by a 20° half-angle cone with nonuniform radiosity and nondiffuse emittance; ϵ/ϵ normal = $0.5 + 0.5 \cos\theta$, $\omega = 1.0$, $\eta_0 = 1.0$, $\beta = 20°$.

As a result of the above considerations, a number of calculations were made as shown in Figs. 2–9. First new values were calculated to correct Bobco's graph $\omega = 0$ as shown in Figs. 2 and 3. The effective shape factor was computed directly also for $\omega = 1$ as shown in Figs. 4 and 5. Values were also obtained for emissivity ratio $0.5 + 0.5 \cos\theta$, where θ is the angle from the plume surface local normal. Such a relation is an approximate fit to the directional emissivity of a slab of scattering particles with a high single scattering albedo.

In line with the preceding remarks Figs. 8 and 9 should be used for a high-albedo, scattering-thick, absorbing-thin plume. Figures 6 and 7 should be used for a high-albedo, scattering-thick, absorbing-thick plume. Figures 4 and 5 apply to a low-albedo, scattering-thick, absorbing-thin plume, and Figs. 2 and 3 apply to a low-albedo, scattering-thick, absorbing-thick plume. As remarked before, a plume which is appreciably absorbing thick should probably not be treated as an opaque surface due to attenuation by colder lightweight particles on the edges of the plume. Figures 8 and 9 are thus considered the most useful of the set.

In order to obtain the irradiation on the receiver, the normal intensity $I_0 = J_0/\pi$ must be known. It is best measured. For an absorbing thin plume an estimate may be made as follows:

$$J_0 = Q_a \tau \sigma T^4$$

where T is the particle temperature (the fusion point) and τ is the geometric optical depth

$$\tau = \Sigma_k (3/4r_k) \overline{(\rho_k \mathrm{D})} / \rho_{\mathrm{solid}}$$

where r_k is the particle radius of the kth sized particle, $\overline{\rho_k \mathrm{D}}$ is the average of the particle density times the plume diameter, and ρ_{solid} is the density of the material forming the particle.

A concluding remark is that shadowing by the rocket nozzle may be accounted for by using shape factor algebra[7] in conjunction with the Bobco conical surface concept.

References

[1] Bobco, R. P., "Radiation from Conical Surfaces with Nonuniform Radiosity," *AIAA Journal*, Vol. 4, No. 3, March 1966, pp. 544–546.

[2] Morizumi, S. J. and Carpenter, H. J., "Thermal Radiation

from the Exhaust Plume of an Aluminized Composite Propellant Rocket," *Journal of Spacecraft and Rockets*, Vol. 1, No. 5, Sept.-Oct. 1964, pp. 501–507.

[3] Edwards, D. K., "Review of Thermal Radiation from TRW INTELSAT III Aluminized Solid Rocket Plume," Rept. 68-3346.1-34, May 13, 1968, TRW Systems Group, Redondo Beach, Calif.

[4] Hottel, H. C. and Sarofim, A. F., *Radiative Transfer*, McGraw-Hill, New York, 1967, p. 381.

[5] Plass, G. N., "Temperature Dependence of Mie Scattering and Absorption Cross Section for Aluminum Oxide," *Applied Optics*, Vol. 4, 1965, pp. 1616–1619.

[6] Chandrasekhar, S., *Radiative Transfer*, Dover Publications, New York, 1960, Chap. V.

[7] Sparrow, E. M. and Cess, R. D., *Radiation Heat Transfer*, Brooks/Cole, Belmont, Calif., 1966, pp. 129–134.

Reply by Author to D. K. Edwards

Richard P. Bobco*
Hughes Aircraft Company, El Segundo, Calif.

The author gratefully acknowledges Dr. Edwards' correction of the shape factor curves in Ref. 1 and the comments on the utility of approximating a metal oxide laden exhaust plume as a conical surface. His enlargement of the concept to introduce an "effective shape factor" should prove useful to thermal designers who must protect spacecraft surfaces from rocket plume radiation.

The analysis of Ref. 1 represented a first, and apparently unsteady, step in attempting to reduce an extremely difficult problem to a level which could be treated by hardware oriented thermal designers. Subsequent studies of the "metal oxide plume problem" are reported in Ref. 2–4, in which the integro-differential equation of radiative transfer is solved by the diffusion-iteration approximation. These studies together with a vastly different problem reported in Ref. 5 support the following conclusions: 1) the emittance from an isothermal, isotropically scattering dispersion is not diffuse (Ref. 2), 2) directional emittance, uniform over a "surface," is analytically equivalent to a surface of nonuniform radiostiy when computing the radiant flux at a remote location (Ref. 5, 2–4), and 3) the value $\omega = 1$ is the best approximation for an isotropically scattering conical plume with an albedo approaching unity and an exit optical-scattering depth of about 3 (Ref. 4).

The author agrees that the reference radiosity J_0 should be a measured quantity rather than computed. However, if J_0 is measured at the exit plane of a rocket nozzle (a common location), the assumption $\omega = 1$ is conservative insofar as the axial decay is more rapid. The exit-plane radiosity is enhanced by radiation from heteropolar gas products that cool rapidly downstream of the exit plane, and the "searchlight" effect of the combustion chamber and nozzle enclosure irradiating particles immediately downstream of the exit plane. Both of these effects become less pronounced several exit diameters downstream of the exit plane. A third influence, that of anisotropic scattering, on the magnitude of ω cannot be assessed properly at this time. The results of an exploratory study of the searchlight effect together with anisotropic scattering are reported by Stockham and Love.[6] However, their cylindrical geometry with constant density is a model of the plume in the vicinity of the exit plane only and does not provide any insight to the magnitude of ω.

Dr. Edwards' product $Q_a\tau$ is an apparent emittance at the exit plane. In Ref. 4 it is identified as the normal emissivity (emittance) at the cone surface. An order of magnitude uncertainty in the imaginary part of the refraction index for fused Al_2O_3 indicates that $0.02 \leq Q_a\tau \leq 0.20$ where $\tau = 3$. This observation, together with the uncertain influence of gaseous radiation and searchlight effect provides additional support for obtaining J_0 from measurements rather than computations.

Finally, the expression for directional emittance, $\epsilon/\epsilon_{\text{normal}} = 0.50\ (1 + \cos\theta)$, appears to be a reasonable fit for Chandrasekhar's H-function for semi-infinite plane dispersions, but the emittance from a conical geometry has a more complicated directional character. Emittance curves for a conical geometry are presented in Ref. 4 and these show both polar and azimuthal dependence on angle. In the plane containing the conical axis, the emittance resembles $\epsilon/\epsilon_{\text{normal}} \approx 1 + \sin\theta$ while in a plane normal to the axis $\epsilon/\epsilon_{\text{normal}} \approx \cos\theta$. However, in view of 1) the equivalence between nondiffuse local emittance and a nonuniform radiosity distribution, and 2) the rapid axial decay of gaseous radiation and the searchlight effect, it appears that Edwards' Figs. 8 and 9 are most useful for thermal design. It should be observed that the combination $\omega = 1$ and $\epsilon = 0.50\ (1 + \cos\theta)$ is roughly equivalent to the choice $\omega = 2$.

Received February 11, 1969.
* Senior Staff Engineer. Associate Fellow AIAA.

References

[1] Bobco, R. P., "Radiation from Conical Surfaces with Nonuniform Radiosity," *AIAA Journal*, Vol. 4, No. 3, March 1966, pp. 544–546.

[2] Edwards, R. H. and Bobco, R. P., "Radiant Heat Transfer from Isothermal Dispersions with Isotropic Scattering," Rept. SSD 50134R, ATL No. 3, Dec. 1965, Hughes Aircraft Co.; abridged version published in *Journal of Heat Transfer, ASME Transactions*, Vol. 89, Ser. C., No. 4, Nov. 1967, pp. 300–308.

[3] Bobco, R. P., "Directional Emissivities from a Two-Dimensional, Absorbing-Scattering Medium: The Semi-Infinite Slab," *Journal of Heat Transfer, ASME Transactions*, Vol. 89, Ser. C, No. 4, Nov. 1967, pp. 313–320.

[4] Bobco, R. P. and Edwards, R. H., "Radiation from an Absorbing Scattering Conical Dispersion with Non-Uniform Density," Rept. SSD 60571R, ATRR No. 21, Dec. 1966, Hughes Aircraft Co.; Rept. abridged version in Paper 68-HT-29, Aug. 1968, American Society of Mechanical Engineers.

[5] Bobco, R. P. and Stensgaard, C. P., Jr., "Solar Incidence Factors for Thermal Analysis of Imperfectly Collimated Radiation in Solar Simulators," *Journal of Spacecraft and Rockets*, Vol. 4, No. 2, Feb. 1967, pp. 236–242.

[6] Stockham, L. W. and Love, T. J., "Monte Carlo Solution of Radiative Heat Transfer from a Finite Cylindrical Cloud of Particles," Paper 68-64, Jan. 1968, AIAA.

Comments on "A Reattachment Criterion for Turbulent Supersonic Separated Flows"

R. H. Page*
Rutgers University, New Brunswick, N. J.

Batham[1] has shown that his reattachment criterion correlates certain experimental data with an empirical constant K. This correlation should be interpreted as one possible correlation in a region of possible solutions. It has been shown[2] that the turbulent pressure coefficient at reattachment depends upon both Reynolds number (initial boundary layer) and Mach number for free reattachments.

Received February 17, 1969.
* Professor and Chairman, Department of Mechanical & Aerospace Engineering. Associate Fellow AIAA.